Exercise 32

If $f(x) = \cos(\ln x^2)$, find $f'(1)$.

Solution

Take the derivative of the function with respect to x by using the chain rule repeatedly.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\cos(\ln x^2) \\ &= -\sin(\ln x^2) \cdot \frac{d}{dx}(\ln x^2) \\ &= -\sin(\ln x^2) \cdot \left[\frac{1}{x^2} \cdot \frac{d}{dx}(x^2)\right] \\ &= -\sin(\ln x^2) \cdot \left[\frac{1}{x^2} \cdot (2x)\right] \\ &= -\sin(\ln x^2) \cdot \left(\frac{2}{x}\right) \\ &= -\frac{2\sin(\ln x^2)}{x} \end{aligned}$$

Now set $x = 1$ to get $f'(1)$.

$$f'(1) = -\frac{2\sin(\ln 1^2)}{1} = -2\sin(0) = 0$$